LITERATI'10

Event Guidelines !!!

Hey folks,

Welcome to the world of numbers. And if you have that extra bit of logic in you, this event would be something you would love to have a look at. Follow the guidelines and rattle your brains through the event to be the best……Or you may very well back out and rest!!!!.....

1. There are two sets of problems.
2. Each team will have to attempt both the sets and the winner would be the team with maximum sum total of both the sets.
3. The maximum marks for every problem is written in bold within brackets.
4. There will be negative marking and the rules regarding this will be launched with the particular set.
5. In case of a tie, the bold questions would carry extra weightage.
6. Further ties would be resolved on the basis of time of submission.
7. The solution should be in .doc, .jpeg or .pdf format.
8. **E-mail your solutions to literati2k10@gmail.com with the subject line "Online Math Submission- set 2".**
9. **Last date of the submission for the solutions of set2 is 30th JANUARY, 2010 BY 5 p.m.**
10. Clearly mention your Name, College name and Contact details with the solution set.

See you at LITERATI'10……

LITERATI'10

Online math challenge

SET-2

... From the bin of progfool!

SECTION-1 (OBJECTIVE)

Rules for the questions 1-7:

(a) More than one choice may be correct.

(b) Any wrong answer will attract penalty of 1/4 of the total credit for that question.

1. Let a coordinate plane AB is given.
A set of 2000 points $\{(a_1, b_1), (a_2, b_2),..., (a_{2000}, b_{2000})\}$ is called an ideal set if $0 \le a_i \le 83$, $0 \le b_i \le 1$ for all i = 1, 2, ..., 2000 and $a_i \neq a_j$ when $i \neq j$.

What will be the largest positive integer P such that, for any ideal set, the interior and boundary of some unit square in the given plane contains exactly P of the points in the set on its interior or its boundary? (3)

OPTIONS
(a) 25
(b) 30
(c) 32
(d) 28

2. Let PQR be a triangle such that $QR \le RP \le PQ$.
Let R and r be the circumradius and inradius of the triangle.
Then the value of QR + RP - 2R - 2r is: (3)

OPTIONS:
(a) Negative if angle R is an acute angle.
(b) Positive if angle R is an obtuse angle.
(c) Zero if angle R is a right angle.
(d) Always Zero.

3. Suppose P be a positive prime for which the following equality holds.
$P^n = a^3+b^3$, where a, b and n are positive integers.

How many such prime numbers are possible? (3)

OPTIONS:
(a) Seven
(b) Five
(c) Six
(d) None of the above

4. We define an infinite sequence of positive integers an F-sequence if every term of this sequence (starting from the 3rd term) equals the sum of the two preceding terms.
Then we ..... decompose the set of all positive integers into a/an ..... number of F-sequences having no common members.

What are the appropriate entries for the given blanks? (3)

OPTIONS:
(a) Can, finite
(b) Can't, finite
(c) Can, infinite
(d) Can't, infinite

5. **There are 2000 congruent triangles, each of area 1.**
These triangles are images of a single triangle under different translations.
Each of these triangles contains the cancroids of all the others.

Then the area of the union of these triangles must be: (4)

OPTIONS:
(a) Less than 19/9
(b) Less than 22/9
(c) less than 29/9
(d) Less than 17/9

6. **A mathematics test paper consists of 5 multiple choice questions, each having 4 different choices; 2000 students take the test, and each student chooses exactly one answer per question.**

   What will be the smallest possible value of n for which it is possible for the student's answer sheets to have the following property:
   among any n of the student's answer sheets, there exist 4 of them among which any two have at most 3 common answers. (4)

   OPTIONS:
   (a) 25
   (b) 35
   (c) 50
   (d) 40

7. How many positive integers P are possible having the property

   $P = (D(P))^2$.
   Here D (P) denotes the number of positive divisors of P. (3)

   OPTIONS:
   (a) 4
   (b) 2
   (c) 6
   (d) 7

SECTION-2 (SUBJECTIVE)

Rules for the questions 8-10:

(a) Give a brief and self-explanatory step by step solution of the problem.

(b) No negative marking in this section.

8. Let we define a property of a positive integer. It is called "monotonic" If its digits in base 10, when read from left to right are in non-decreasing order.

   Prove that for each n ε N, there exists an n-digit monotonic number which is a perfect square. (3)

9. **Does there exist a function F: N→N such that**

F (F(n-1))= F(n+1) – F(n)
For all n ≥ 2?

(HINT: prove by contradiction) **(4)**

10. In a convex quadrilateral PQRS, $\angle$QRS= $\angle$RSP. The bisector of angle PQR intersects RS at point E.

Prove that $\angle$PEQ=90° If and only if PQ=PS+QR. (3)