

# LITERATI'10

## Event Guidelines

Hey folks,

Welcome to the world of numbers. And if you have that extra bit of logic in you, this event would be something you would love to have a look at. Follow the guidelines and rattle your brains through the event to be the best... Or you may very well back out and rest!

1. It is a team event. A team may consist of not more than two members.
2. There will be two sets of problems.
3. Each team will have to attempt both the sets and the winner would be the team with maximum sum total of both the sets.
4. Each set will comprise of two sections and each section will contain five problems.
5. The maximum marks for every problem is written in bold within brackets.
6. There will be negative marking and the rules regarding this will be launched with the particular set.
7. In case of a tie, the bold questions would carry extra weightage.
8. Further ties would be resolved on the basis of time of submission.
9. The solution should be in .doc, .jpeg or .pdf format.
10. **E-mail your solutions to literati2k10@gmail.com with the subject line "Online Math Submission-set 1". Please ensure that the subject line reads exactly as what has been mentioned.**
11. **Last date of the submission for the solutions of set1 is 23<sup>rd</sup> JANUARY, 2010 BY 11 PM.**
12. Clearly mention your Name, College name and Contact details with the solution set.

See you at LITERATI'10..

# LITERATI'10

## Online math challenge

### SET-1

... From the bin of progfool!

#### SECTION-1 (OBJECTIVE)

Rules for the questions 1-5:

- (a) Only one answer is correct.
- (b) Any wrong answer will attract penalty of  $1/4$  of the total credit for that question.

1. How many triplets of natural numbers  $(a, b, c)$  are possible, such that  $a, b$  and  $c$  are in geometric progression, and  $a + b + c = 111$ . (2)

OPTIONS:

- (a) 7
- (b) 5
- (c) 6
- (d) 4
- (e) Can't be calculated.

2. Let ABCD be a square of side 1. P and Q are points on AB and BC such that  $\angle PDQ = 45^\circ$ . What will be the perimeter of  $\Delta PBQ$ ? (2)

OPTIONS:

- (a) 2.67
- (b) 2.0
- (c) 2.5
- (d) 3.0
- (e) None

3. Suppose that the real numbers  $S_1, S_2, \dots, S_{100}$  satisfy

$$S_1 \geq S_2 \geq \dots \geq S_{100} \geq 0,$$

$$S_1 + S_2 \leq 100,$$

$$\text{and } S_3 + S_4 + \dots + S_{100} \leq 100:$$

Let  $M$  be the maximum possible value of  $S_1^2 + S_2^2 + \dots + S_{100}^2$ , and  $T$  be the total possible sequences  $S_1, S_2, \dots, S_{100}$  which achieve this maximum.

Then the values of  $M$  and  $T$  are: (2)

OPTIONS:

(a)  $M = 10000, T = 2$

(b)  $M = 10000, T = 3$

(c)  $M = 20000, T = 1$

(d)  $M = 20000, T = 3$

(e)  $M = 10000, T = 4$

4. Suppose a real number  $P$  is such that

$$[(1/2)p] + [(1/3)p] + [(1/5)p] = p$$

Here,  $[x]$  denotes the greatest integer that is less than or equal to  $x$ , where  $x$  is a real number.

How many solutions for  $p$  are possible? (2)

OPTIONS:

(a) 28

(b) 32

(c) 31

(d) 30

5. How many ways are there to express 1 000000 as a product of exactly three integers greater than 1?

(For the purpose of this problem,  $abc$  is not considered different from  $bac$ , etc.) (3)

OPTIONS:

(a) 114

(b) 139

(c) 123

(d) 132

## SECTION-2 (FILL THE GAPS)

Rules for the questions 6-10:

(a) Give only answers of the questions, no need of description.

(b) No negative marking in this section.

6. If  $[x [x [x [x]]]] = 88$ .

The notation  $[x]$  means: "the least integer which is not less than  $x$ " and  $x$  is a real number.

Let  $m$  be the total no. of possible solutions of  $x$ . Then the  $m$  is.....(3)

7. If  $4x^2 - 40[x] + 51 = 0$ .

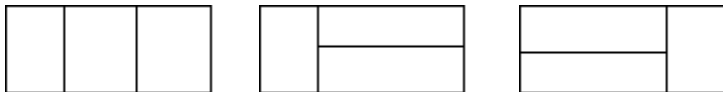
Here, if  $x$  is a real number, then  $[x]$  denotes the greatest integer that is less than or equal to  $x$ .

Then the number of all possible real solutions of  $x$  is..... (3)

8. A number  $X_n$  of the form  $1010101\dots1$  has  $n$  ones. The total number of all  $n$  such that  $X_n$  is prime is..... (3)

9. Non-congruent triangles with integer sides and perimeter 1999 are to be constructed? Total number of such possible triangles is..... (3)

10. There are three ways to tile a  $2 \times 3$  rectangle using  $2 \times 1$  tiles: (4)



Let  $a_n$  be the no. of ways to tile a  $4 \times n$  rectangle using  $2 \times 1$  tiles.

$a_n$  is divisible by 2 if and only if  $a_n$  is divisible by.....

(Fill the gap with an appropriate number)